

高等数学微积分公式大全

一、基本导数公式

$$(1) (c)' = 0 \quad (2) x^\mu = \mu x^{\mu-1} \quad (3) (\sin x)' = \cos x$$

$$(4) (\cos x)' = -\sin x \quad (5) (\tan x)' = \sec^2 x \quad (6) (\cot x)' = -\csc^2 x$$

$$(7) (\sec x)' = \sec x \cdot \tan x \quad (8) (\csc x)' = -\csc x \cdot \cot x$$

$$(9) (e^x)' = e^x \quad (10) (a^x)' = a^x \ln a \quad (11) (\ln x)' = \frac{1}{x}$$

$$(12) (\log_a x)' = \frac{1}{x \ln a} \quad (13) (\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (14) (\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$

$$(15) (\arctan x)' = \frac{1}{1+x^2} \quad (16) (\operatorname{arccot} x)' = -\frac{1}{1+x^2} \quad (17) (x)' = 1 \quad (18) (\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

二、导数的四则运算法则

$$(u \pm v)' = u' \pm v' \quad (uv)' = u'v + uv' \quad \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

三、高阶导数的运算法则

$$(1) [u(x) \pm v(x)]^{(n)} = u^{(n)}(x) \pm v^{(n)}(x) \quad (2) [cu(x)]^{(n)} = cu^{(n)}(x)$$

$$(3) [u(ax+b)]^{(n)} = a^n u^{(n)}(ax+b) \quad (4) [u(x) \cdot v(x)]^{(n)} = \sum_{k=0}^n C_n^k u^{(n-k)}(x) v^{(k)}(x)$$

四、基本初等函数的 n 阶导数公式

$$(1) (x^n)^{(n)} = n! \quad (2) (e^{ax+b})^{(n)} = a^n \cdot e^{ax+b} \quad (3) (a^x)^{(n)} = a^x \ln^n a$$

$$(4) [\sin(ax+b)]^{(n)} = a^n \sin\left(ax+b+n \cdot \frac{\pi}{2}\right) \quad (5) [\cos(ax+b)]^{(n)} = a^n \cos\left(ax+b+n \cdot \frac{\pi}{2}\right)$$

$$(6) \left(\frac{1}{ax+b}\right)^{(n)} = (-1)^n \frac{a^n \cdot n!}{(ax+b)^{n+1}} \quad (7) [\ln(ax+b)]^{(n)} = (-1)^{n-1} \frac{a^n \cdot (n-1)!}{(ax+b)^n}$$

五、微分公式与微分运算法则

$$(1) d(c) = 0 \quad (2) d(x^\mu) = \mu x^{\mu-1} dx \quad (3) d(\sin x) = \cos x dx$$

$$^{(4)} d(\cos x) = -\sin x dx \quad ^{(5)} d(\tan x) = \sec^2 x dx \quad ^{(6)} d(\cot x) = -\csc^2 x dx$$

$$^{(7)} d(\sec x) = \sec x \cdot \tan x dx \quad ^{(8)} d(\csc x) = -\csc x \cdot \cot x dx$$

$$^{(9)} d(e^x) = e^x dx \quad ^{(10)} d(a^x) = a^x \ln a dx \quad ^{(11)} d(\ln x) = \frac{1}{x} dx$$

$$^{(12)} d(\log_a x) = \frac{1}{x \ln a} dx \quad ^{(13)} d(\arcsin x) = \frac{1}{\sqrt{1-x^2}} dx \quad ^{(14)} d(\arccos x) = -\frac{1}{\sqrt{1-x^2}} dx$$

$$^{(15)} d(\arctan x) = \frac{1}{1+x^2} dx \quad ^{(16)} d(\operatorname{arccot} x) = -\frac{1}{1+x^2} dx$$

六、微分运算法则

$$^{(1)} d(u \pm v) = du \pm dv \quad ^{(2)} d(cu) = cdu$$

$$^{(3)} d(uv) = vdu + u dv \quad ^{(4)} d\left(\frac{u}{v}\right) = \frac{vdu - u dv}{v^2}$$

七、基本积分公式

$$^{(1)} \int k dx = kx + c \quad ^{(2)} \int x^\mu dx = \frac{x^{\mu+1}}{\mu+1} + c \quad ^{(3)} \int \frac{dx}{x} = \ln|x| + c$$

$$^{(4)} \int a^x dx = \frac{a^x}{\ln a} + c \quad ^{(5)} \int e^x dx = e^x + c \quad ^{(6)} \int \cos x dx = \sin x + c$$

$$^{(7)} \int \sin x dx = -\cos x + c \quad ^{(8)} \int \frac{1}{\cos^2 x} dx = \int \sec^2 x dx = \tan x + c$$

$$^{(9)} \int \frac{1}{\sin^2 x} dx = \int \csc^2 x dx = -\cot x + c \quad ^{(10)} \int \frac{1}{1+x^2} dx = \arctan x + c$$

$$^{(11)} \int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + c$$

八、补充积分公式

$$\int \tan x dx = -\ln|\cos x| + c \quad \int \cot x dx = \ln|\sin x| + c$$

$$\int \sec x dx = \ln|\sec x + \tan x| + c \quad \int \csc x dx = \ln|\csc x - \cot x| + c$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \arctan \frac{x}{a} + c \quad \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin \frac{x}{a} + c \quad \int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln \left| x + \sqrt{x^2 \pm a^2} \right| + c$$

九、下列常用凑微分公式

积分型	换元公式
$\int f(ax+b)dx = \frac{1}{a} \int f(ax+b)d(ax+b)$	$u = ax+b$
$\int f(x^\mu)x^{\mu-1}dx = \frac{1}{\mu} \int f(x^\mu)d(x^\mu)$	$u = x^\mu$
$\int f(\ln x) \cdot \frac{1}{x} dx = \int f(\ln x)d(\ln x)$	$u = \ln x$
$\int f(e^x) \cdot e^x dx = \int f(e^x)d(e^x)$	$u = e^x$
$\int f(a^x) \cdot a^x dx = \frac{1}{\ln a} \int f(a^x)d(a^x)$	$u = a^x$
$\int f(\sin x) \cdot \cos x dx = \int f(\sin x)d(\sin x)$	$u = \sin x$
$\int f(\cos x) \cdot \sin x dx = -\int f(\cos x)d(\cos x)$	$u = \cos x$
$\int f(\tan x) \cdot \sec^2 x dx = \int f(\tan x)d(\tan x)$	$u = \tan x$
$\int f(\cot x) \cdot \csc^2 x dx = \int f(\cot x)d(\cot x)$	$u = \cot x$
$\int f(\arctan x) \cdot \frac{1}{1+x^2} dx = \int f(\arctan x)d(\arctan x)$	$u = \arctan x$
$\int f(\arcsin x) \cdot \frac{1}{\sqrt{1-x^2}} dx = \int f(\arcsin x)d(\arcsin x)$	$u = \arcsin x$

十、分部积分法公式

(1)形如 $\int x^n e^{ax} dx$, 令 $u = x^n$, $dv = e^{ax} dx$

形如 $\int x^n \sin x dx$ 令 $u = x^n$, $dv = \sin x dx$

形如 $\int x^n \cos x dx$ 令 $u = x^n$, $dv = \cos x dx$

(2) 形如 $\int x^n \arctan x dx$, 令 $u = \arctan x$, $dv = x^n dx$

形如 $\int x^n \ln x dx$, 令 $u = \ln x$, $dv = x^n dx$

(3) 形如 $\int e^{ax} \sin x dx$, $\int e^{ax} \cos x dx$ 令 $u = e^{ax}$, $\sin x, \cos x$ 均可。

十一、第二换元积分法中的三角换元公式

(1) $\sqrt{a^2 - x^2}$ $x = a \sin t$ (2) $\sqrt{a^2 + x^2}$ $x = a \tan t$ (3) $\sqrt{x^2 - a^2}$ $x = a \sec t$

【特殊角的三角函数值】

(1) $\sin 0 = 0$ (2) $\sin \frac{\pi}{6} = \frac{1}{2}$ (3) $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$ (4) $\sin \frac{\pi}{2} = 1$ (5) $\sin \pi = 0$

(1) $\cos 0 = 1$ (2) $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$ (3) $\cos \frac{\pi}{3} = \frac{1}{2}$ (4) $\cos \frac{\pi}{2} = 0$ (5) $\cos \pi = -1$

(1) $\tan 0 = 0$ (2) $\tan \frac{\pi}{6} = \frac{\sqrt{3}}{3}$ (3) $\tan \frac{\pi}{3} = \sqrt{3}$ (4) $\tan \frac{\pi}{2}$ 不存在 (5) $\tan \pi = 0$

(1) $\cot 0$ 不存在 (2) $\cot \frac{\pi}{6} = \sqrt{3}$ (3) $\cot \frac{\pi}{3} = \frac{\sqrt{3}}{3}$ (4) $\cot \frac{\pi}{2} = 0$ (5) $\cot \pi$ 不存在

十二、重要公式

(1) $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ (2) $\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e$ (3) $\lim_{n \rightarrow \infty} \sqrt[n]{a} (a > 0) = 1$

(4) $\lim_{n \rightarrow \infty} \sqrt[n]{n} = 1$ (5) $\lim_{x \rightarrow \infty} \arctan x = \frac{\pi}{2}$ (6) $\lim_{x \rightarrow -\infty} \arctan x = -\frac{\pi}{2}$

(7) $\lim_{x \rightarrow \infty} \operatorname{arccot} x = 0$ (8) $\lim_{x \rightarrow -\infty} \operatorname{arccot} x = \pi$ (9) $\lim_{x \rightarrow -\infty} e^x = 0$

(10) $\lim_{x \rightarrow +\infty} e^x = \infty$ (11) $\lim_{x \rightarrow 0^+} x^x = 1$

(12) $\lim_{x \rightarrow \infty} \frac{a_0 x^n + a_1 x^{n-1} + \cdots + a_n}{b_0 x^m + b_1 x^{m-1} + \cdots + b_m} = \begin{cases} \frac{a_0}{b_0} & n = m \\ 0 & n < m \\ \infty & n > m \end{cases}$ (系数不为 0 的情况)

十三、下列常用等价无穷小关系 ($x \rightarrow 0$)

$\sin x \sim x$ $\tan x \sim x$ $\arcsin x \sim x$ $\arctan x \sim x$ $1 - \cos x \sim \frac{1}{2} x^2$

$$\ln(1+x) \sim x \quad e^x - 1 \sim x \quad a^x - 1 \sim x \ln a \quad (1+x)^{\theta} - 1 \sim \theta x$$

十四、三角函数公式

1.两角和公式

$$\sin(A+B) = \sin A \cos B + \cos A \sin B \quad \sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B \quad \cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\begin{aligned} \tan(A+B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} & \tan(A-B) &= \frac{\tan A - \tan B}{1 + \tan A \tan B} \\ \cot(A+B) &= \frac{\cot A \cdot \cot B - 1}{\cot B + \cot A} & \cot(A-B) &= \frac{\cot A \cdot \cot B + 1}{\cot B - \cot A} \end{aligned}$$

2.二倍角公式

$$\sin 2A = 2 \sin A \cos A \quad \cos 2A = \cos^2 A - \sin^2 A = 1 - 2 \sin^2 A = 2 \cos^2 A - 1$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

3.半角公式

$$\sin \frac{A}{2} = \sqrt{\frac{1 - \cos A}{2}} \quad \cos \frac{A}{2} = \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \frac{A}{2} = \sqrt{\frac{1 - \cos A}{1 + \cos A}} = \frac{\sin A}{1 + \cos A} \quad \cot \frac{A}{2} = \sqrt{\frac{1 + \cos A}{1 - \cos A}} = \frac{\sin A}{1 - \cos A}$$

4.和差化积公式

$$\sin a + \sin b = 2 \sin \frac{a+b}{2} \cdot \cos \frac{a-b}{2} \quad \sin a - \sin b = 2 \cos \frac{a+b}{2} \cdot \sin \frac{a-b}{2}$$

$$\cos a + \cos b = 2 \cos \frac{a+b}{2} \cdot \cos \frac{a-b}{2} \quad \cos a - \cos b = -2 \sin \frac{a+b}{2} \cdot \sin \frac{a-b}{2}$$

$$\tan a + \tan b = \frac{\sin(a+b)}{\cos a \cdot \cos b}$$

5.积化和差公式

$$\sin a \sin b = -\frac{1}{2} [\cos(a+b) - \cos(a-b)] \quad \cos a \cos b = \frac{1}{2} [\cos(a+b) + \cos(a-b)]$$

$$\sin a \cos b = \frac{1}{2} [\sin(a+b) + \sin(a-b)] \quad \cos a \sin b = \frac{1}{2} [\sin(a+b) - \sin(a-b)]$$

6.万能公式

$$\sin a = \frac{2 \tan \frac{a}{2}}{1 + \tan^2 \frac{a}{2}} \quad \cos a = \frac{1 - \tan^2 \frac{a}{2}}{1 + \tan^2 \frac{a}{2}} \quad \tan a = \frac{2 \tan \frac{a}{2}}{1 - \tan^2 \frac{a}{2}}$$

7.平方关系

$$\sin^2 x + \cos^2 x = 1 \quad \sec^2 x - \tan^2 x = 1 \quad \csc^2 x - \cot^2 x = 1$$

8.倒数关系

$$\tan x \cdot \cot x = 1 \quad \sec x \cdot \cos x = 1 \quad \csc x \cdot \sin x = 1$$

9.商数关系

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

十五、几种常见的微分方程

1.可分离变量的微分方程： $\frac{dy}{dx} = f(x)g(y)$ ， $\int f_1(x)g_1(y)dx + \int f_2(x)g_2(y)dy = 0$

2.齐次微分方程： $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$

3.一阶线性非齐次微分方程： $\frac{dy}{dx} + p(x)y = Q(x)$ 解为：

$$y = e^{-\int p(x)dx} \left[\int Q(x)e^{\int p(x)dx} dx + c \right]$$